Fast GPU Fitting of Kinetic Models for Dynamic PET

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MICCAI 2010 – Workshop on High-Performance Medical Image Computing for Image-Assisted Clinical Intervention and Decision Making

September 24th, 2010

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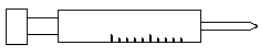
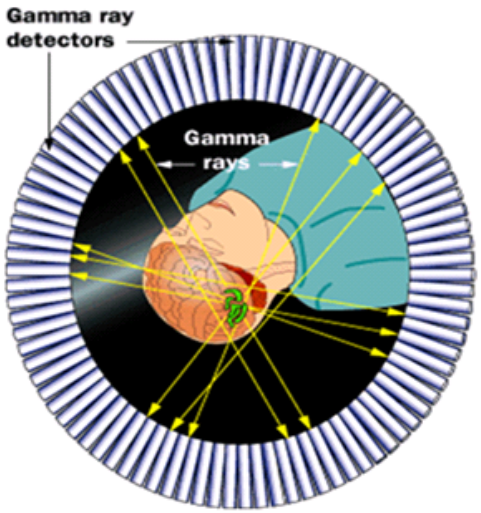
Introduction to Dynamic PET





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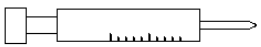
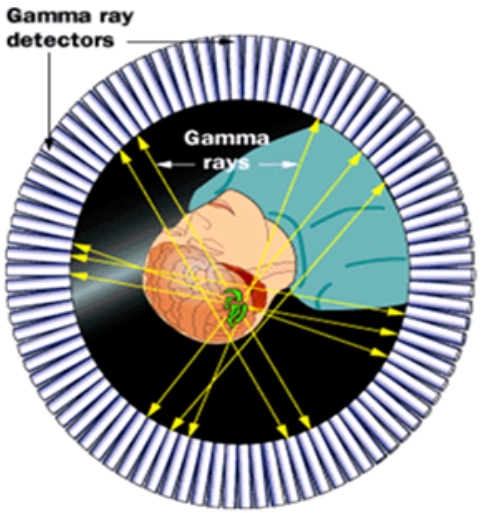
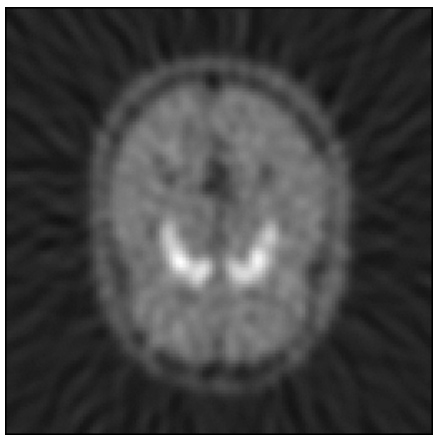
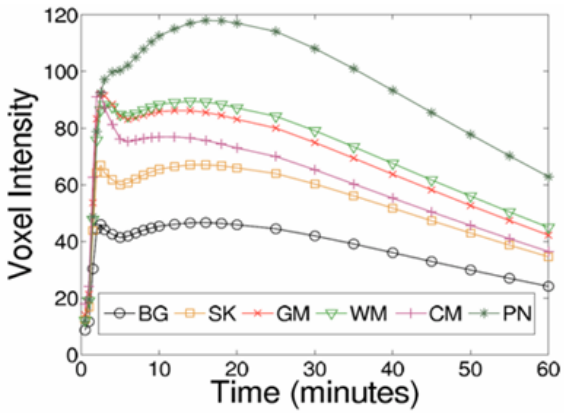
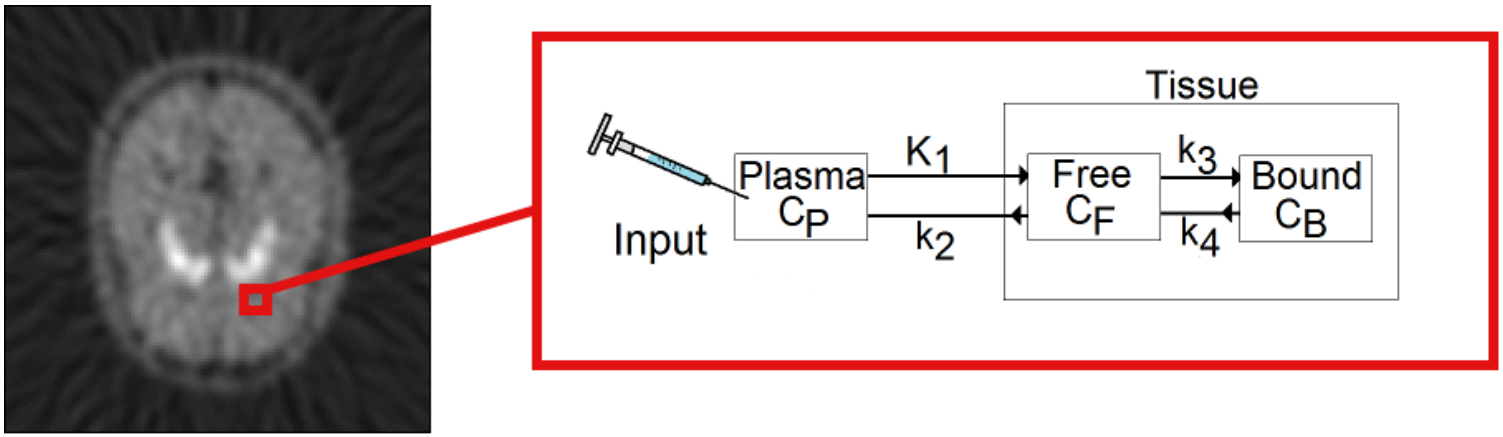








Kinetic Modelling



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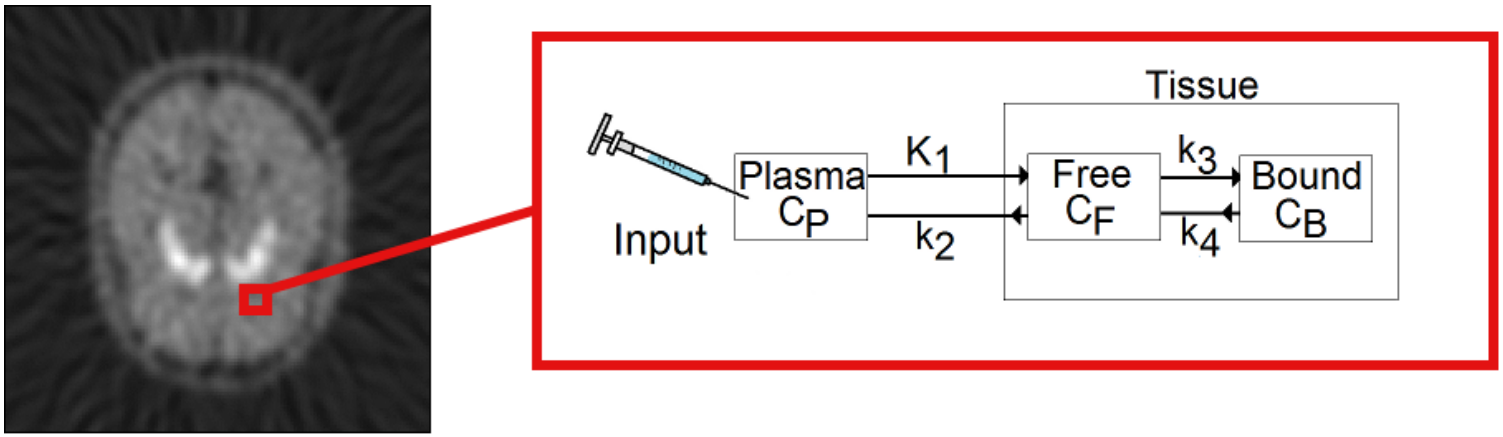

$$\begin{aligned} \frac{dC_F}{dt} &= K_1 C_P - k_2 C_F - k_3 C_F + k_4 C_B - \lambda C_F, \\ \frac{dC_B}{dt} &= k_3 C_F - k_4 C_B - \lambda C_F. \end{aligned} \tag{1}$$

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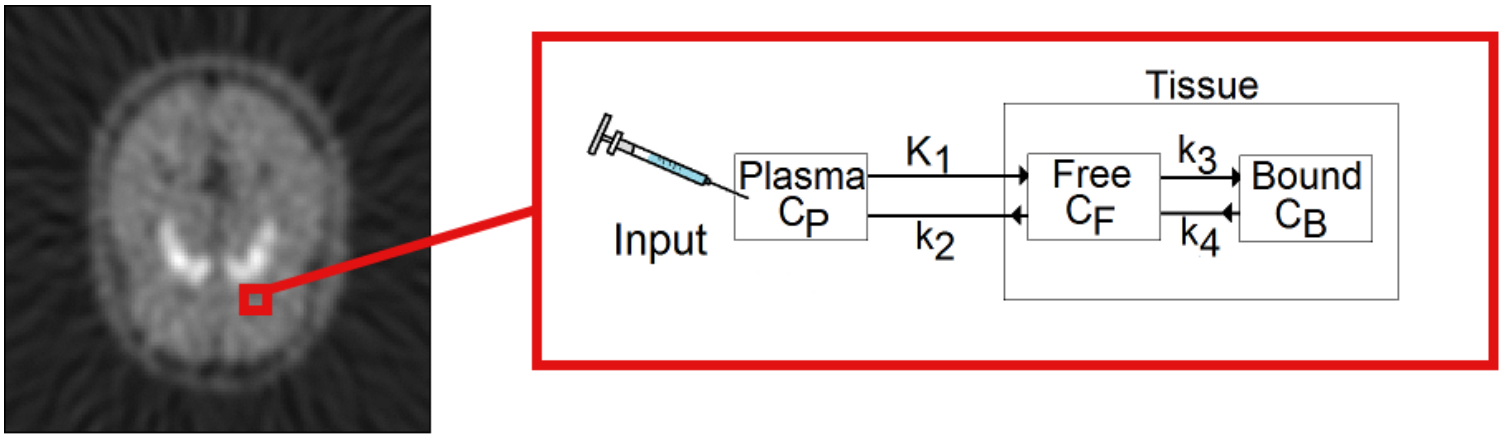

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$$GMR = \frac{k_1 k_3}{k_2 + k_3} \propto \text{metabolism} \tag{2}$$

Fitting Kinetic Models

According to Gunn et al., 2001 [2], the solution to these ODEs is:

$$\underbrace{O(t)}_{\substack{\text{Output}\\ \text{function}}} = \underbrace{\sum_{i=1}^{C} C_i(t)}_{\substack{\text{Sum of}\\ \text{concentrations}}} = \left[\underbrace{I_f}_{\substack{\text{Input}\\ \text{function}}} * \underbrace{\left(\sum_{i=1}^{C} \varphi_i e^{-\theta_i t} \right)}_{\substack{\text{Response}\\ \text{function}}} \right](t). \tag{3}$$

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Recover kinetic parameters at each voxel x by minimization:

$$\min_{\theta,\phi} \sum_{t=0}^{T} \left[O(t) - I(x,t) \right]^2 \tag{4}$$

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- Kinetic models are nonlinear (exponential terms)
- Simple optimization is insufficient: typically, Levenberg-Marquardt algorithm is used
- Available software, COMKAT (Muzik et al., 2001 [1]) compartmental modelling tookit, takes seconds per fitting of a single TAC.

Observations

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- Performs numerical convolution of finely sampled input function
- $\Rightarrow$ With an analytic input function, the convolution can be solved analytically!
- Computes numerical approximation of gradient and jacobian of fitting function
- $\Rightarrow$ With an analytic input function, the derivatives can be computed exactly!

Analytic Kinetic Fitting

Analytic input function (D. Feng et al., 2003 [3]):

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The derivatives are defined exactly as well, for use in the Levenberg-Marquardt algorithm!

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- Levenberg-Marquardt is an iterative algorithm
- Can implement algorithm in CUDA, and store algorithm state for each thread
- Threads are kept synchronized by performing algorithm steps until all fittings in a block complete.

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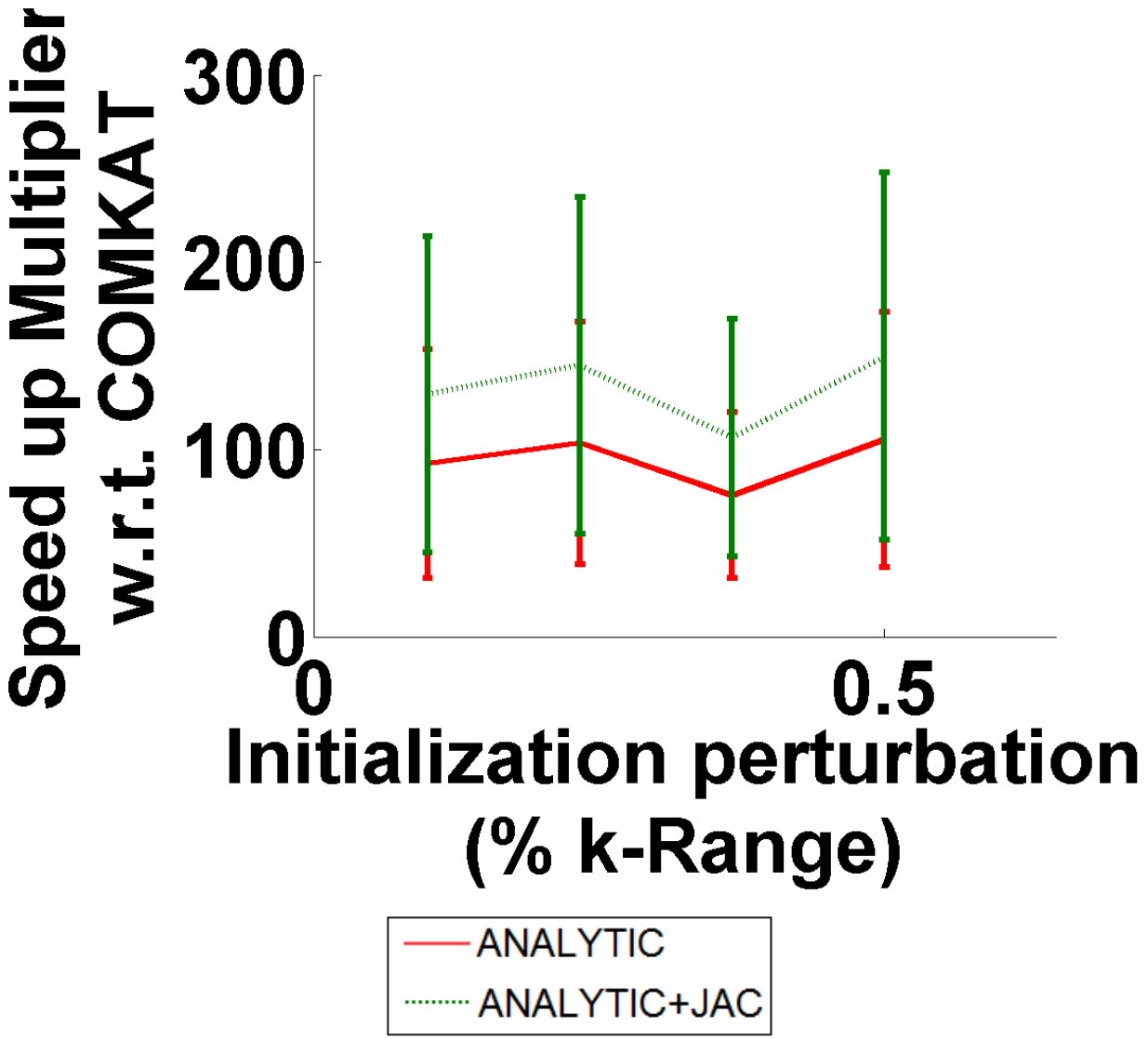
- Four levels of each parameter were tested, with the other parameter fixed at a reasonable value.
- For each level tested, 20 repetitions were performed with different realizations of noise and perturbation.

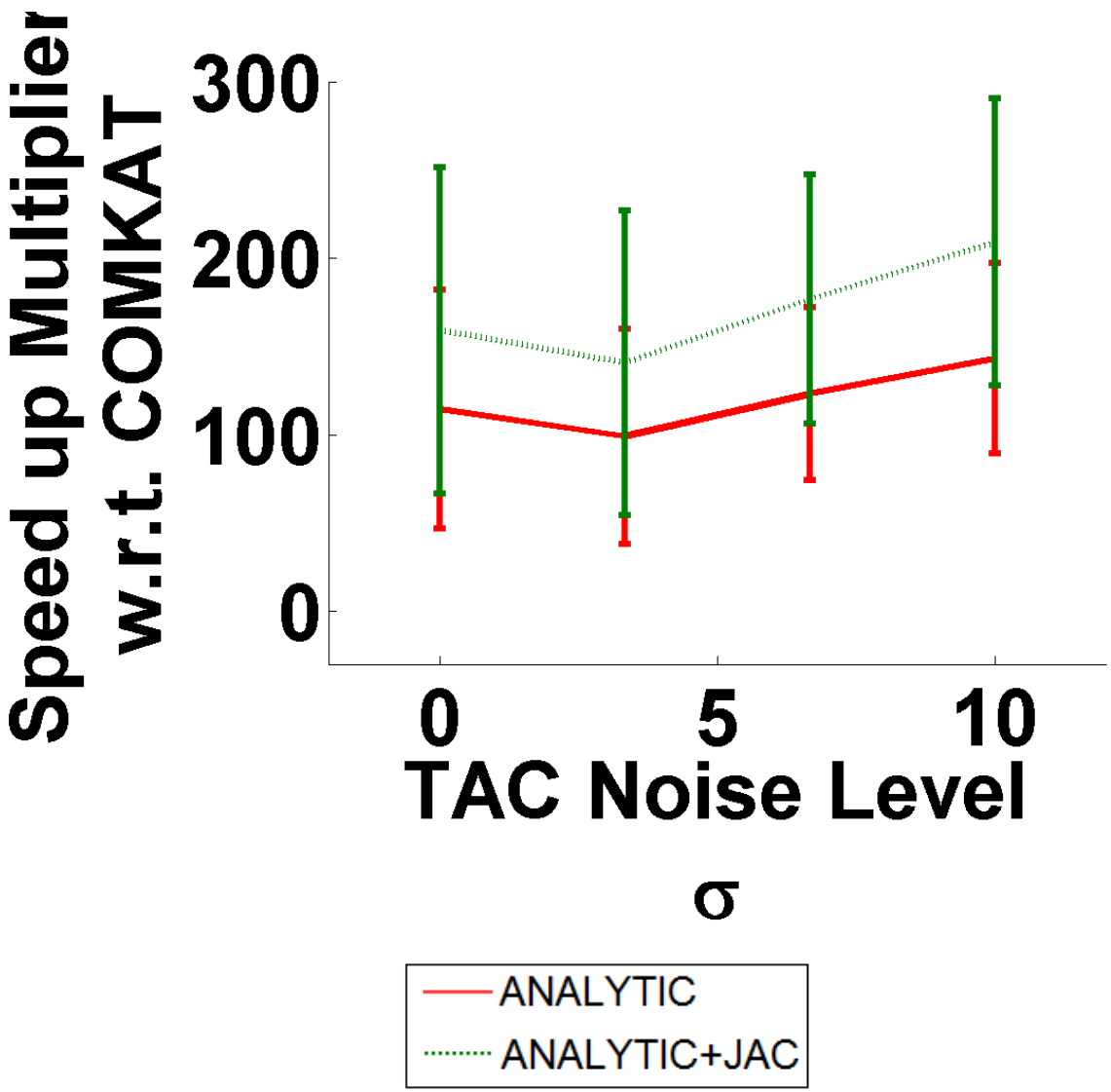
Compared Methods

- ANALYTIC: Analytic output function, approximated derivatives
- ANALYTIC+JAC: Analytic output function, analytic derivatives
- COMKAT: COMKAT fitting tool
- CUDA: Parallel GPU implementation of ANALYTIC+JAC

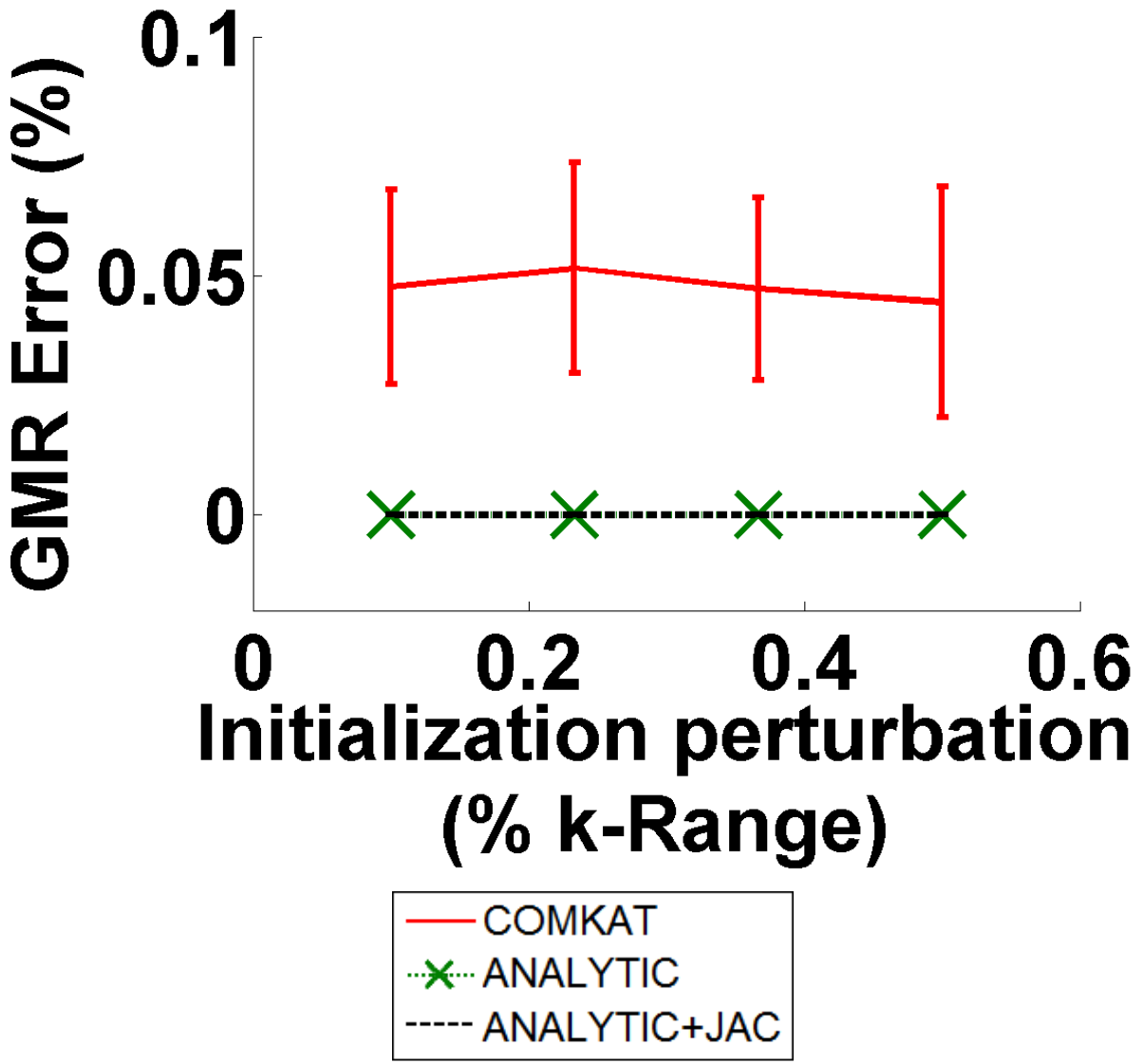
Speedup vs. Perturbation



Speedup vs. Noise



Error vs. Perturbation



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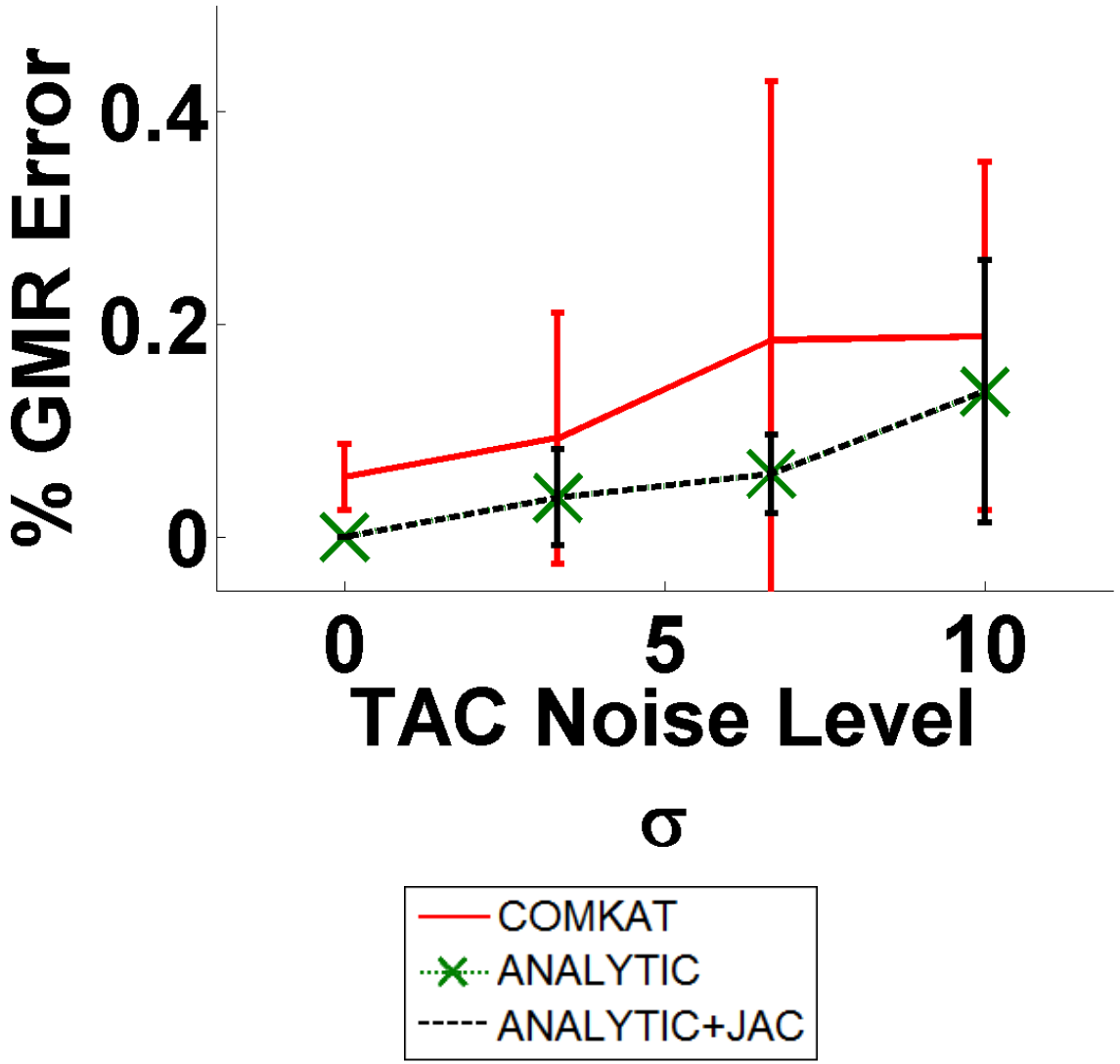

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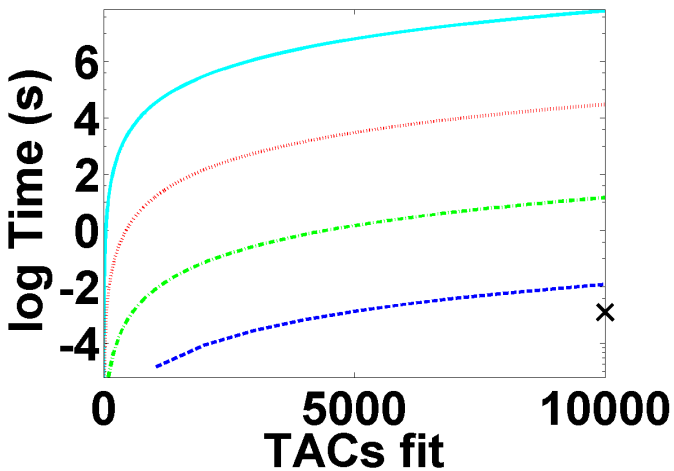
- Level of parallelism was varied from 1 to 10,000
- Execution time was recorded at each level
- Prediction: For each factor 10 increase in parallelism, speed of fitting should increase by 10

Parallelism Results



	Parallelism	Speed (TAC/s)
———	1	45
·····	10	450
-·-·-	100	4,505.6
- - -	1,000	33,758.4
×	10,000	74,906.4

Conclusion

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- Analytic methods demonstrate speed-up of approximately $100\times$ over COMKAT, with equivalent accuracy.
- Performing fitting in parallel with CUDA furthered increased performance to $130,000\times$.
- This allows a typical PET volume of $128 \times 128 \times 63$ to be fit in approximately 13.8 seconds, as compared to 21.5 days using COMKAT.

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Future Work

- Investigate parallel spatial regularization of fitting
- Explore application to other modalities (DCE-MRI)

References

Thank you for your time! Questions?

Presented by: Ahmed Saad - aasaad@cs.sfu.ca

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References

1. R. Muzic et al., COMKAT: compartment model kinetic analysis tool. *J. Nucl. Med.*, 42(4):636-645, 2001.
2. R.N. Gunn et al., PET compartmental models. *J. Cereb. Blood Flow and Metab.*, 21(6):635-652, 2001.
3. D. Feng et al., Models for computer simulation of input functions for tracer kinetic modeling with PET. *Int. Bio. Comp.*, 32:95-110, 1993.